

# Graduation Numeracy Assessment

## Sample Assessment Student-Choice Component: Scoring Guide and Student Exemplars



BRITISH  
COLUMBIA

Ministry of  
Education

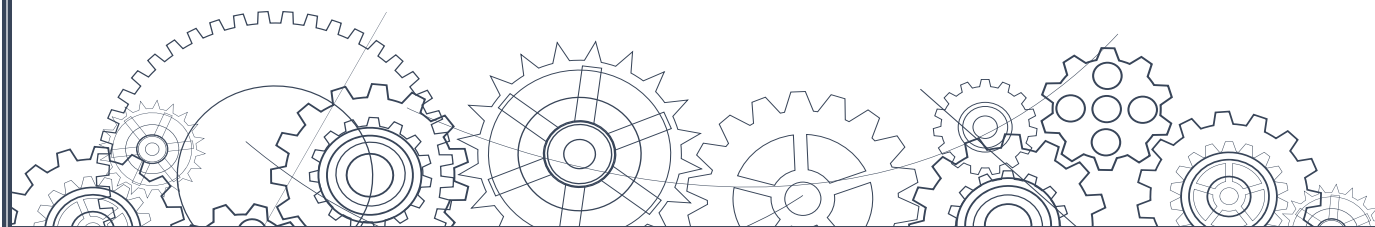
# 2017





# Table of Contents

Rubric and Elaborations .....	1
Water Use .....	2
<i>Question</i> .....	2
<i>Summary of Requirements for a Level 4</i> .....	4
<i>Possible Solutions</i> .....	5
<i>Exemplars</i> .....	7
Pit Houses .....	15
<i>Question</i> .....	15
<i>Summary of Requirements for a Level 4</i> .....	16
<i>Possible Solutions</i> .....	17
<i>Exemplars</i> .....	21
Video Game Company .....	30
<i>Question</i> .....	30
<i>Summary of Requirements for a Level 4</i> .....	31
<i>Possible Solutions</i> .....	31
<i>Exemplars</i> .....	34
Forest Fires .....	46
<i>Question</i> .....	46
<i>Summary of Requirements for a Level 4</i> .....	49
<i>Possible Solutions</i> .....	50
<i>Exemplars</i> .....	52





# Rubric and Elaborations



## Rubric

	1	2	3	4
Snapshot	<p>Student demonstrates a limited understanding of the situation. Approach is ineffective. Mathematical solution contains fundamental errors. Reasoning or evidence is absent or incomplete.</p>	<p>Student demonstrates a basic understanding of the situation. Approach is difficult to follow. Mathematical solution contains some errors. Reasoning or evidence is lacking to some degree.</p>	<p>Student demonstrates an effective understanding of the situation. Approach is based on effective mathematical reasoning and principles and can generally be followed. Mathematical solution may contain minor errors or may not be completely evaluated in context. Reasoning or evidence contains minor inconsistencies.</p>	<p>Student demonstrates an advanced understanding of the situation. Approach is based on advanced mathematical reasoning and principles and can be easily followed. Mathematical solution satisfies the situation criteria and is evaluated in context. Reasoning and evidence is clear and well presented.</p>
	<p><b>NR</b> No response (answer page is blank).</p>	<p><b>0</b> Data simply copied from the problem. Picture, work or solution is unrelated to the problem. An incorrect mathematical solution with no work shown. Inappropriate response (work contains profanity, inappropriate diagram or language). Work is erased.</p>		



## Elaborations

	1	2	3	4
Interpret	<ul style="list-style-type: none"> <li>• Demonstrates limited reasoning skills in interpreting the situation.</li> <li>• Makes fundamental errors when making decisions about relevant information given in text, symbols, or graphical forms.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates basic reasoning skills in interpreting the situation.</li> <li>• Makes some errors when making decisions about relevant information given in text, symbols, or graphical forms.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates effective reasoning skills in interpreting the situation.</li> <li>• Makes minor errors when making decisions about relevant information given in text, symbols, or graphical forms.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates advanced reasoning skills in interpreting the situation.</li> <li>• Makes no errors when making decisions about relevant information given in text, symbols, or graphical forms.</li> </ul>
Apply	<ul style="list-style-type: none"> <li>• Demonstrates limited success in selecting and applying an effective approach for the situation.</li> <li>• Creates algebraic expressions or equations with fundamental errors in logic.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates some success in selecting and applying an effective approach for the situation.</li> <li>• Creates algebraic expressions or equations with some errors in logic.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates success in selecting and applying an effective approach for the situation.</li> <li>• Creates algebraic expressions or equations with minor errors in logic.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates success in selecting and applying a sophisticated approach for the situation.</li> <li>• Creates algebraic expressions or equations with no errors in logic.</li> </ul>
Solve	<ul style="list-style-type: none"> <li>• Demonstrates limited use of mathematical concepts and skills involved in the situation.</li> <li>• Mathematical solution is incomplete or incorrect, with fundamental calculation errors.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates basic use of mathematical concepts and skills involved in the situation.</li> <li>• Mathematical solution is incorrect, with some calculation errors.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates effective use of mathematical concepts and skills involved in the situation.</li> <li>• Mathematical solution is incorrect, with minor calculation errors.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates advanced use of mathematical concepts and skills involved in the situation.</li> <li>• Mathematical solution is correct, with no calculation errors.</li> </ul>
Analyze	<ul style="list-style-type: none"> <li>• No attempt to assess the reasonableness of solution, or attempt contains fundamental errors in reasoning.</li> <li>• Demonstrates limited ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.</li> </ul>	<ul style="list-style-type: none"> <li>• Assesses the reasonableness of solution, but contains some errors in reasoning.</li> <li>• Demonstrates basic ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.</li> </ul>	<ul style="list-style-type: none"> <li>• Effective analysis about the reasonableness of solution, but may contain minor errors in reasoning.</li> <li>• Demonstrates effective ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.</li> </ul>	<ul style="list-style-type: none"> <li>• Insightful analysis about the reasonableness of solution.</li> <li>• Demonstrates advanced ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.</li> </ul>
Communicate	<ul style="list-style-type: none"> <li>• Demonstrates limited ability to communicate using mathematical language (e.g., graphs, symbols).</li> <li>• Presents limited or irrelevant evidence for the solution.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates basic ability to communicate using mathematical language (e.g., graphs, symbols).</li> <li>• Presents some evidence for the solution; solution may have inconsistencies or be difficult to follow.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates effective ability to communicate using mathematical language (e.g., graphs, symbols).</li> <li>• Presents effective evidence for the solution using logical arguments.</li> </ul>	<ul style="list-style-type: none"> <li>• Demonstrates advanced ability to communicate using mathematical language (e.g., graphs, symbols).</li> <li>• Presents sophisticated evidence for the solution using logical arguments.</li> </ul>

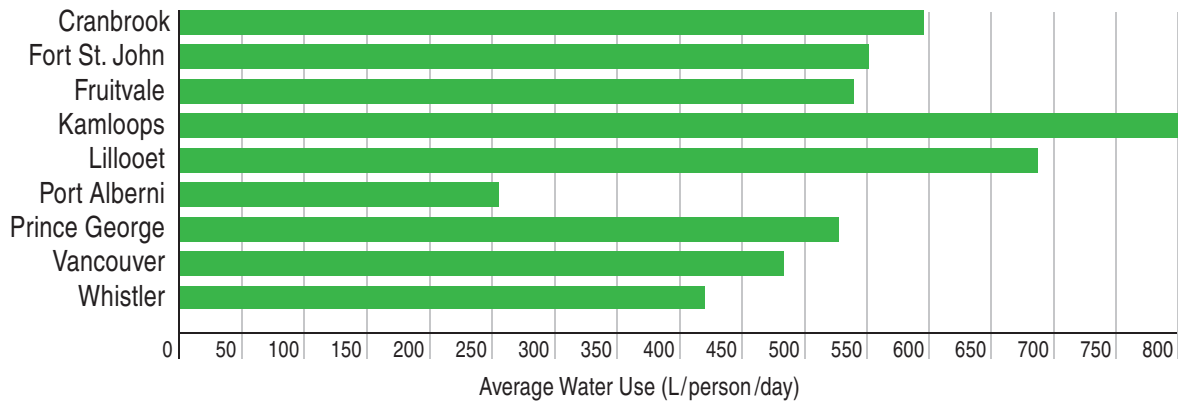
Rubric and elaborations will be further refined following the first assessment administration and will be available in the spring of 2018.

A newspaper headline grabs your attention.








# DAILY NEWS

## Water use skyrockets

Recent studies have predicted water shortages.



### Indoor Water Use with Conventional Appliances and Fixtures

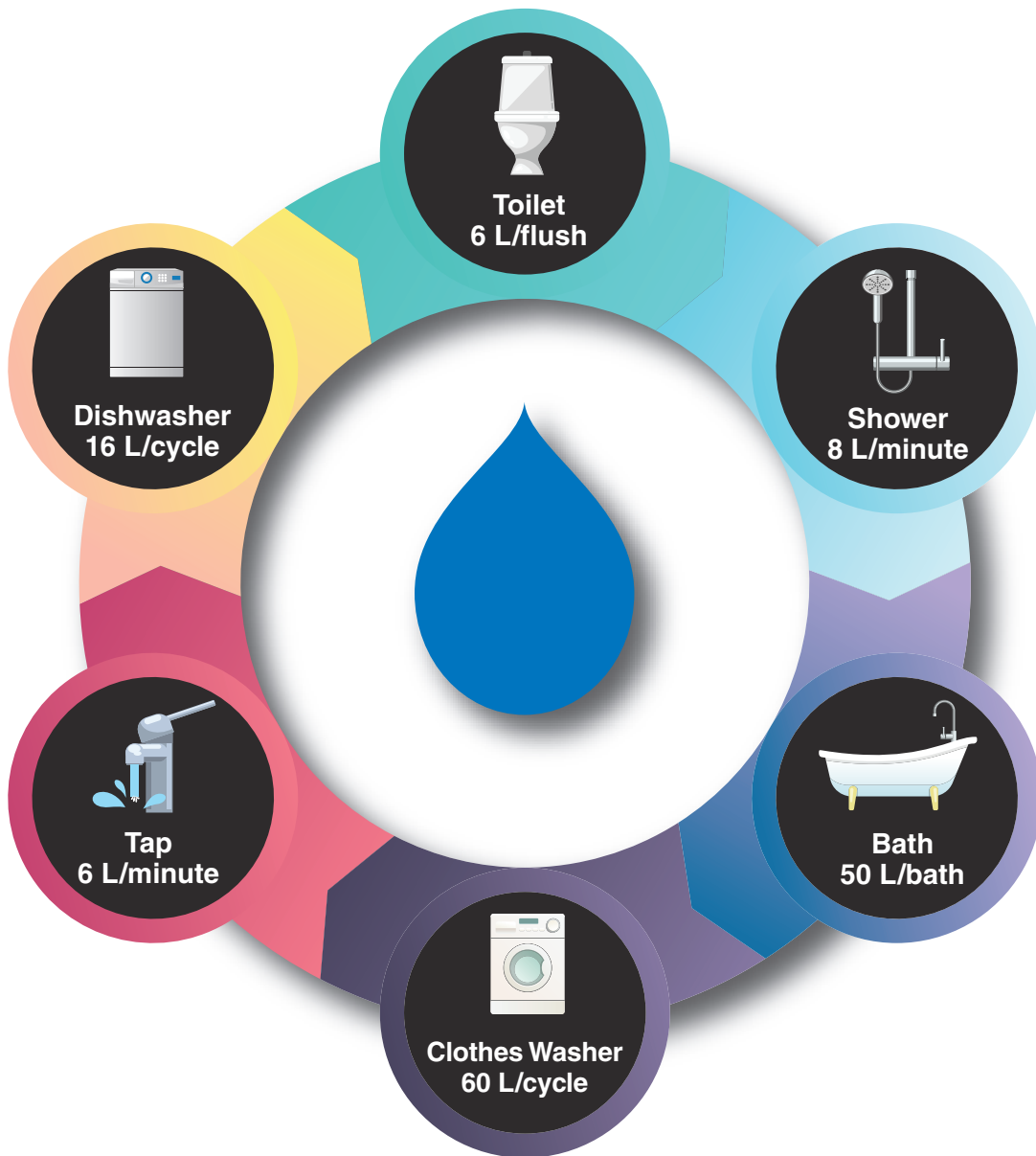
	Toilet	13 L/flush		Tap	8 L/minute
	Shower	10 L/minute		Dishwasher	40 L/cycle
	Bath	100 L/bath		Leaks	3 L/day
	Clothes Washer	150 L/cycle			

\$2.75 Our office: 16, 1st Street Volume 51 / Est. 1987



High-efficiency appliances and fixtures can help reduce the amount of water we use.

### Indoor Water Use with High-Efficiency Appliances and Fixtures



13. You want to reduce your personal water use to 1050 L/week. You install high-efficiency appliances and fixtures, and change your water-use habits.

Plan a water budget for yourself for 1 week that meets this goal using the high-efficiency appliances and fixtures.

Explain and justify your solution.

You must use everything in the table below at least once in the week.

High-Efficiency Appliances and Fixtures	
	Shower and/or bath
	Toilet
	Tap
	Dishwasher
	Clothes Washer

This question is to be answered on paper.

### Summary of Requirements for a Level 4:

- ▶ Use less than 1050 L/week
- ▶ Use each of the items in the table (shower or bath, toilet, tap, dishwasher, clothes washer) at least once
- ▶ Be realistic (e.g., cannot flush toilet once per week)
- ▶ Can do a daily budget and then show that it is being replicated daily for the week, **or** can do a full seven-day budget with water usage varying each day
- ▶ Provide evidence to justify budget
- ▶ Communicate solution in context



## Water Use

There are many possible solutions; a couple solutions are given.

### Solution 1

#### Daily Water Usage

Activity/Appliance	Daily Usage	Total	Explanation
Shower	5 min/day $5 \times 8 \text{ L/min} = 40 \text{ L}$	40 L	One quick shower per day is all that is needed
Toilet	3 flushes/day $3 \times 6 \text{ L} = 18 \text{ L}$	18 L	Assuming you work or go to school during the day you only average using the toilet 3x/day
Tap	10 min/day $10 \times 6 \text{ L/min} = 60 \text{ L}$	60 L	After using washroom, and once for dishes that can't go in dishwasher
Dishwasher	Once every second day	8L	One load of dishes per two days
Clothes Washer	Two loads/week (/7)	17 L	Only use a light and dark load per week
<b>TOTAL</b>		143 L/day	

$143 \text{ L/day} \times 7 \text{ days} = 1001 \text{ L}$  for the week, which is under the limit of 1050 L per week.

## Solution 2

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Shower/ bath	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L
Toilet	3 × 6 L = 18 L	3 × 6 L = 18 L	3 × 6 L = 18 L	3 × 6 L = 18 L	3 × 6 L = 18 L	5 × 6 L = 30 L	5 × 6 L = 30 L
Tap	8 min × 6 L/min = 48 L	10 min × 6 L/min = 60 L	8 min × 6 L/min = 48 L	10 min × 6 L/min = 60 L	8 min × 6 L/min = 48 L	10 min × 6 L/min = 60 L	8 min × 6 L/min = 48 L
Dishwasher	1 × 16 L = 16 L		1 × 16 L = 16 L		1 × 16 L = 16 L		1 × 16 L = 16 L
Clothes washer		1 cycle = 60 L				1 cycle = 60 L	
<b>Total</b>	<b>122L</b>	<b>178L</b>	<b>122L</b>	<b>118L</b>	<b>122L</b>	<b>190L</b>	<b>134L</b>

Weekly total:  $122 + 178 + 122 + 118 + 122 + 190 + 134 = 986$  L

Weekly total is 986 L which is well under the total of 1050 L per week needed. This leaves room for any unexpected water usage.

Showers were taken daily. Toilet usage was increased on the weekend as I would not be using the facility at school as I do during the week.

The dishwasher is used every second day, so on the days it doesn't run there are an extra 2 minutes of tap usage to accommodate having to wash a couple of dishes by hand.

Two loads of clothes washing per week are enough to have one load of lights and one load of darks.

This keeps the water usage under the limit of 1050 L per week.

## Exemplar #1 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

Let's say we use 4 L of toilet twice a day =

$$6 \cdot 2 = 12 \text{ L}$$

$$150 - 12 = 138 \text{ L left}$$

We have 138 L left.

We shower 4 min a day =

$$4 \cdot 8 = 32$$

$$138 - 32 = 106$$

We have one cloth wash every other day so =

in one week we have 3 cloth washes =

$$3 \cdot 60 = 180 \text{ L}$$

$$180 : 7 = 25.7 \text{ L}$$

So we use 25.7 L per day

$$106 - 25.7 \text{ L} = 80.3$$

We use the taps at least 10 min each day =

$$10 \cdot 6 = 60$$

$$80.3 - 60 = 20.3$$

We have 20.3 L left.

We use the dishwasher once a day =

$$16 \text{ L}$$

$$20.3 - 16 = 4.3$$

We have exactly 4.3 L left so that in case of an emergency we can still use 4.3 L without passing our limit.

## Water Use

### Exemplar #2 – Score: 4

- Work shows an advanced understanding of the situation
- Appropriate strategy implemented
- Correct mathematical solution
- Evaluates mathematical solution in context
- Communication is clear, detailed, and organized

$150\text{L/day} \cdot 7\text{ days} = 1050\text{L/week (maximum)}$

1 shower per day =  $8\text{L} \cdot 11\text{mins} \cdot 7\text{ days} = 616\text{L}$

Clothes washer twice per week =  $120\text{L}$

Dishwasher 3 times per week =  $16\text{L} \cdot 3\text{ uses} = 48\text{L}$

Toilet 4 times a day (maximum) =  $6\text{L} \cdot 4\text{ uses} \cdot 7\text{ days} = 168\text{L}$

Tap  $\frac{1}{2}$  min per hand wash 4 times per day + 1 min for cooking per day =  $(\frac{1}{2} \cdot 4 + 1) \cdot 7 = 21\text{L}$

Total:

616L  
120L  
48L +  
168L  
21L

---

973L/week

$1050\text{L/week} > 973\text{L/week}$  so I managed to use less than an average of 1050 L in my weekly budget.

### Exemplar #3 – Score: 3

- Work shows an effective understanding of the situation
- Appropriate strategy implemented
- Correct mathematical solution
- Incomplete justification (did not adjust solution to meet criteria)
- Communication is clear

Handwritten student work on a grid background. At the top right, the calculation  $1W = 7 \times 150 = 1050 L$  is written, with a downward arrow pointing to the text "Per week". Below this, a list of daily water uses is calculated and summed:

$$\begin{aligned} 5 \text{ Flushes a day} &= 30L \times 7 = \underline{210L} \\ 10 \text{ min tap a day} &= 60 \times 7 = \underline{420L} \\ 5 \text{ showers 5 min} &= 40 \times 5 = \underline{200L} \\ 1 \text{ load of laundry} &= \underline{60L} \\ \text{Dishwasher per day} &= \underline{112L} \\ \text{Bath} &= \underline{50L} \\ &= 1052 L \end{aligned}$$

## Water Use

### Exemplar #4 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Incomplete justification (all calculations not shown)
- ✓ Communication is clear and organized

By Day	Why frequencies chosen	
4 minutes a day - 30L a day	one shower a day, for only 4 minutes	shower / bath
2 flushes a day, 20L	avg. person only needs to flush the toilet twice	toilet
15 minutes a day, 30L a day	The bathroom tap can be conserved to 2 minutes leaving 3 for the kitchen	taps (bathroom and kitchen)
1 cycle a day, 10L a day	The dishwasher only needs to be run at the end of the day	dishwasher
1 cycle a day, 60L	The laundry needs to be done at the end of the day ensuring only one cycle	clothes washer
overall consumption in a week = 1050		By reducing shower time and faucet time one can use 150L of water a day, baths are not needed
overall consumption in a day = 150		

## Water Use

### Exemplar #5 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected but not followed through
- ✓ Incomplete mathematical solution
- ✓ Incomplete justification (all calculations not shown)
- ✓ Information is not clearly communicated

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Showers	1 Bath / 80L	∅	1 Bath	∅	1 Bath	∅	1 Bath
Bath							
Toilet	18L / 3 flush	3 flush	3 flush	3 flush	3 flush	3 flush	3 flush
Taps	13.6 min / 81.6L	9.3 min	13.6 min	9.3 min	13.6 min	9.3 min	13.6
Dish-washer	∅	16L	∅	16L	∅	16L	∅
Clothes washer	∅	60L	∅	60L	∅	60L	∅

## Water Use

### Exemplar #6 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected but not followed through
- ✓ Solution is not reasonable (e.g., 1 minute showers)
- ✓ Incomplete justification
- ✓ Information is not clearly communicated

	L
shower(s) and/or bath(s)	8L
Toilet	24L
Faucet (Bath/Kitchen)	42L
Dishwasher	16L
Clothes Washer	60L

Total 150L

~~Shower(s)~~  
 Toilets  
 Faucets  
 Dishwasher  
 CW

150

taps/toilets  
 used frequent.  
 (faucets used more  
 usually)



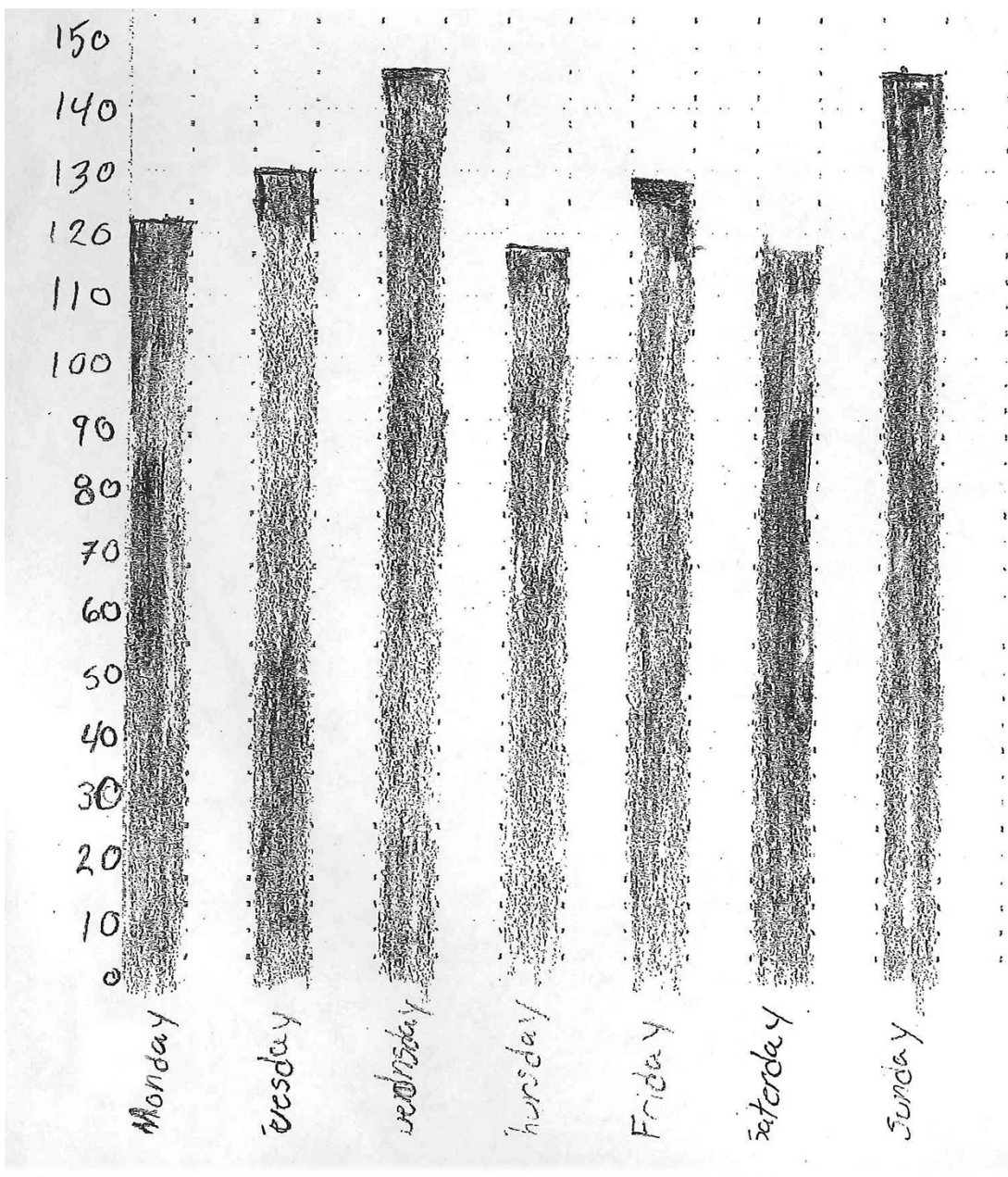
### Exemplar #7 – Score: 1

- Work shows a limited understanding of the situation
- Limited strategy implemented
- No solution
- Justification is not reasonable
- Information is somewhat communicated

- Don't use bath(s) or shower at the same time
- Don't use dishwasher if you don't have much dishes to wash
- Use clothes washer only if you have the right amount of dirty laundry to wash.

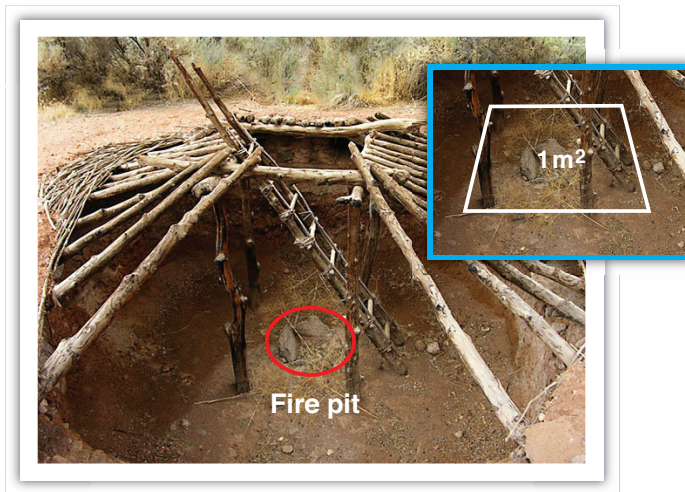
## Exemplar #8 – Score: 1

- Work shows a limited understanding of the situation
- Unclear strategy
- Solution not explained in context
- No justification
- Information is not clearly communicated



## Understanding Our Past: Pit Houses

Archaeologists study artifacts and monuments to understand past cultures. In the interior of British Columbia, First Peoples lived in circular homes, called pit houses. Pit houses varied in size, depending on how many people lived in the home.



### Approximate Floor Area Required

Family

Area: 10 m<sup>2</sup>

Couple

Area: 7 m<sup>2</sup>

Adult

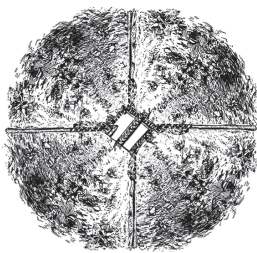
Area: 4 m<sup>2</sup>

Fire Pit and Ladder

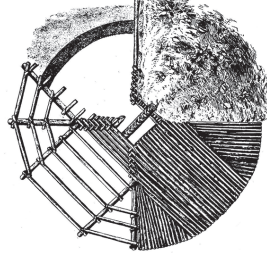
Area: 1 m<sup>2</sup>

### Overhead Views

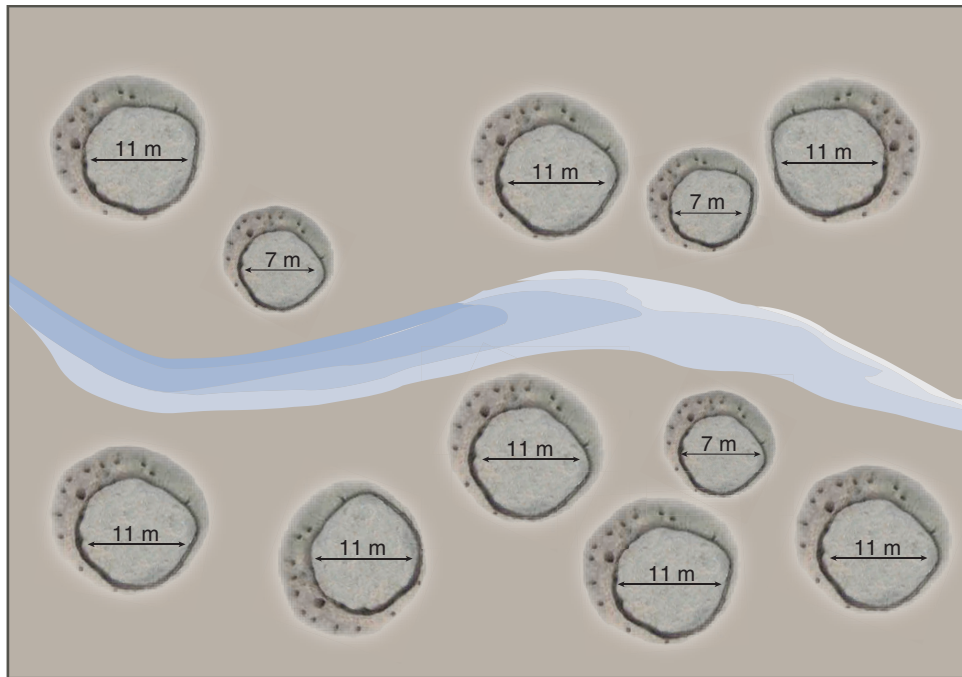
Covered



Structure



13. Archaeologists examine the remains of a village of pit houses. Impressions of where the pit houses were located are still visible. The diagram below shows the remains of a village with a number of pit houses.



Estimate the number of people that could have lived in this village. State any assumptions made.

Explain and justify your solution.

This question is to be answered on paper.

### **Summary of Requirements for a Level 4:**

- ▶ State initial assumptions
- ▶ Calculate area of small and large pit houses
- ▶ Estimate number of people
- ▶ Provide evidence to justify estimate
- ▶ Communicate solution in context

## Pit Houses

There are many possible solutions; several solutions are given.

### Solution 1

Assumptions:

- 1 family consisting of 2 adults and 2 children requires 10 m<sup>2</sup> of living space
- ladder and fire pit require an area of 1 m<sup>2</sup> per pit house
- the same configuration of people live in each of the small and large pit houses
- all pit houses are occupied to their full capacity

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi(3.5)^2$$

$$A \approx 38.48 \text{ m}^2$$

Area available for living:

$$38.48 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 37 \text{ m}^2$$

Area of a large pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi(5.5)^2$$

$$A \approx 95.03 \text{ m}^2$$

Area available for living:

$$95.03 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 94 \text{ m}^2$$

**Number of people who lived in a small pit house:**

$$\frac{37 \text{ m}^2}{10 \text{ m}^2/\text{family}} \approx 3 \text{ families}$$

Area required for 3 families:  $3(10 \text{ m}^2) = 30 \text{ m}^2$

$$37 \text{ m}^2 - 30 \text{ m}^2 = 7 \text{ m}^2 \text{ left over}$$

Area required for 1 couple = 7 m<sup>2</sup>

Total number of people who lived in a small pit house:  $3(4) + 1(2) = 14$

Since there are 3 small pit houses:  $3(14) = 42$

**Number of people who lived in a large pit house:**

$$\frac{94 \text{ m}^2}{10 \text{ m}^2/\text{family}} \approx 9 \text{ families}$$

Area required for 9 families:  $9(10 \text{ m}^2) = 90 \text{ m}^2$

$$94 \text{ m}^2 - 90 \text{ m}^2 = 4 \text{ m}^2 \text{ left over}$$

Area required for 1 adult = 4 m<sup>2</sup>

Total number of people who lived in a large pit house:  $9(4) + 1 = 37$

Since there are 8 large pit houses:  $8(37) = 296$

Total number of people:  $42 + 296 = 338$

The estimated number of people who lived in the village is 338. This is reasonable under the above-mentioned assumptions. This solution also assumes that people are evenly distributed amongst the pit houses and that the maximum living space within each pit house is fully utilized.

## Solution 2

Assumptions:

- 1 adult needs 4 m<sup>2</sup> of living space
- 1 couple needs 7 m<sup>2</sup> of living space
- 1 family consisting of 2 adults and 2 children requires 10 m<sup>2</sup> of living space
- ladder and fire pit require an area 1 m<sup>2</sup> per pit house
- the same configuration of people live in each of the small and large pit houses
- the maximum amount of living space in each pit house is used

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi (3.5)^2$$

$$A \approx 38.48 \text{ m}^2$$

Area available for living:

$$38.48 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 37 \text{ m}^2$$

Area of a large pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi (5.5)^2$$

$$A \approx 95.03 \text{ m}^2$$

Area available for living:

$$95.03 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 94 \text{ m}^2$$

**Strategy: Guess and check:**

**Number of people living in small pit houses:**

$$\text{Area required for 2 families: } (2)(10 \text{ m}^2) = 20 \text{ m}^2$$

$$\text{Area required for 4 adults: } (4)(4 \text{ m}^2) = 16 \text{ m}^2$$

Area required is  $20 \text{ m}^2 + 16 \text{ m}^2 = 36 \text{ m}^2$ , which is less than  $37 \text{ m}^2$  that is available.

$$\text{Number of people: } (2)(4) + 4 = 12 \text{ people per house}$$

$$\therefore (3)(12) = 36 \text{ people lived in the 3 small pit houses.}$$

**Number of people living in large pit houses:**

$$\text{Area required for 6 families: } (6)(10 \text{ m}^2) = 60 \text{ m}^2$$

$$\text{Area required for 3 adults: } (3)(4 \text{ m}^2) = 12 \text{ m}^2$$

$$\text{Area required for 3 couples: } (3)(7 \text{ m}^2) = 21 \text{ m}^2$$

Area required is  $60 \text{ m}^2 + 12 \text{ m}^2 + 21 \text{ m}^2 = 93 \text{ m}^2$ , which is less than  $94 \text{ m}^2$  that is available.

$$\text{Number of people: } (6)(4) + 3 + (3)(2) = 33 \text{ people per house}$$

$$\therefore (8)(33) = 264 \text{ people lived in the 8 large pit houses.}$$

$$36 + 264 = 300 \text{ people}$$

The estimated number of people living in the village is 300. This is a reasonable estimate under the above-mentioned assumptions. However, some families may have more children than 2, thus requiring more space. On the other hand, some families may have had only one child thus requiring less space.

### Solution 3

Assumptions:

- 1 adult needs 4 m<sup>2</sup> of living space
- 1 couple needs 7 m<sup>2</sup> of living space
- 1 family consisting of two adults and two children requires 10 m<sup>2</sup> of living space
- ladder and fire pit require an area of 1 m<sup>2</sup> per pit house
- different configurations of people live in each of the small pit houses and a few of the large pit houses

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi (3.5)^2$$

$$A \approx 38.48 \text{ m}^2$$

Area available for living:

$$38.48 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 37 \text{ m}^2$$

Area of a large pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi (5.5)^2$$

$$A \approx 95.03 \text{ m}^2$$

Area available for living:

$$95.03 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 94 \text{ m}^2$$

#### Number of people living in the 3 small pit houses:

Pit House 1:

$$\text{Area required for 2 families + 4 adults: } (2)(10 \text{ m}^2) + (4)(4 \text{ m}^2) = 36 \text{ m}^2$$

$$(2)(4) + 4 = 12 \text{ people}$$

Pit House 2:

$$\text{Area required for 5 couples: } (5)(7 \text{ m}^2) = 35 \text{ m}^2$$

$$(5)(2) = 10 \text{ people}$$

Pit House 3:

$$\text{Area required for 1 family + 2 couples + 3 adults: } (1)(10 \text{ m}^2) + (2)(7 \text{ m}^2) + (3)(4 \text{ m}^2) = 36 \text{ m}^2$$

$$(1)(4) + (2)(2) + 3 = 11 \text{ people}$$

Total number of people who lived in the small pit houses:  $12 + 10 + 11 = 33$  people.

#### Strategy: Guess and Check

#### Number of people living in the 11 large pit houses:

5 large pit houses occupied by families only – 9 families in each house

$$\text{Area required for 9 families: } (9)(10 \text{ m}^2) = 90 \text{ m}^2$$

$$(9)(4) = 36 \text{ people in one large pit house} \quad (36)(5) = 180 \text{ people in 5 large pit houses}$$

3 large pit houses occupied by 11 adults and 7 couples:

$$\text{Area required for 11 adults and 7 couples: } (11)(4 \text{ m}^2) + (7)(7 \text{ m}^2) = 44 \text{ m}^2 + 49 \text{ m}^2 = 93 \text{ m}^2$$

$$11 + (7)(2) = 25 \text{ people} \quad (25)(3) = 75 \text{ people}$$

Total number of people who lived in the large pit houses:  $180 + 75 = 255$  people.  $33 + 255 = 288$  people

The estimated number of people living in the village is 288. This is a reasonable estimate under the above-mentioned assumptions. However, some families may have more children than 2, thus requiring more space. On the other hand, some families may have had only one child, thus requiring less space.

### Solution 4

#### Assumptions

- 1 adult needs 4 m<sup>2</sup> of living space
- 1 couple needs 7 m<sup>2</sup> of living space
- 1 family consisting of 2 adults and 2 children requires 10 m<sup>2</sup> of living space
- ladder and fire pit require an area of 1 m<sup>2</sup> per pit house
- considering the total area of living space across all pit houses rather than by individual pit house

There are 11 pit houses – 3 small with diameter of 7 m and 8 large ones with diameter of 11 m.

Area of a small pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi (3.5)^2$$

$$A \approx 38.48 \text{ m}^2$$

Area available for living:

$$38.48 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 37 \text{ m}^2$$

Area of a large pit house:

$$A = \pi r^2, \quad r = \frac{d}{2}$$

$$A = \pi (5.5)^2$$

$$A \approx 95.03 \text{ m}^2$$

Area available for living:

$$95.03 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} \approx 94 \text{ m}^2$$

$$\text{Total area available for living: } (3)(37 \text{ m}^2) + (8)(94 \text{ m}^2) = 863 \text{ m}^2$$

**Number of people living in the pit houses:**

$$\frac{863 \text{ m}^2}{10 \text{ m}^2/\text{family}} \approx 86 \text{ families}$$

∴ 86 families can live in this area

$$\text{Area required for 86 families: } 86(10 \text{ m}^2) = 860 \text{ m}^2$$

$$863 \text{ m}^2 - 860 \text{ m}^2 = 3 \text{ m}^2 \text{ left over}$$

3 m<sup>2</sup> which is not enough room for a single adult or couple to live

$$\text{Total number of people: } 86(4) = 344 \text{ people}$$

Therefore, my estimate would be 344 people lived in the village.

The estimate of 344 people is reasonable under the above-mentioned assumptions. However it is also likely that there were couples and single adults living in this village. Considering the total area of living space across all pit houses rather than by individual pit house made my calculations faster with fewer steps. However, this assumption may not properly reflect the real-life scenario if the houses have a considerable amount of space unoccupied.



Exemplar #1 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

Assumptions:

- families need  $10\text{m}^2$  living space
- Couples need  $7\text{m}^2$  living space
- Adults need  $4\text{m}^2$  living space
- there will be a combination of families, couples and adults in the community
- fire/ladder requires  $1\text{m}^2$

Small pit house

$$A = \pi r^2 \quad A = \pi(3.5)^2 \quad A = 38.48\text{m}^2$$

$$38\text{m}^2 - 1\text{m}^2 \text{ for fire/ladder} = \sim 37\text{m}^2 \text{ of living space}$$

- Families will be the most efficient use of space, so  $\frac{37\text{m}^2}{10\text{m}^2 \text{ for family}} = 3 \text{ families plus } 7\text{m}^2 \text{ left of space}$

$7\text{m}^2$  is enough room for a couple  
 so a small pit house can fit 3 families and 1 couple  
 which is 14 people

There are 3 small pit houses.  $3 \times 14 = 42$  people

continued on next page

Large pit house area

$$A = \pi r^2 \quad A = \pi (3.5)^2 \quad A = 95.03 \text{ m}^2$$

$$95 \text{ m}^2 - 1 \text{ m}^2 \text{ for firepit} = 94 \text{ m}^2$$

If we assume that about half of the pit houses are for families and the other half are for adults and couples, then we can assume about 5 families live in the pit house, taking up  $50 \text{ m}^2$  living space.

$$94 \text{ m}^2 - 50 \text{ m}^2 = 44 \text{ m}^2 \text{ left over}$$

$$5 \text{ couples} \times 7 \text{ m}^2 \text{ space} = 35 \text{ m}^2$$

$44 \text{ m}^2 - 35 \text{ m}^2 = 9 \text{ m}^2$  left which is enough space for 2 adults with  $1 \text{ m}^2$  left.

Therefore 1 large pit house can hold

5 families + 5 couples + 2 adults

$$5(4) + 5(2) + 2 =$$

$$20 + 10 + 2 = 32 \text{ people}$$

$$8 \text{ large pit houses} \times 32 \text{ people} = 256 \text{ people}$$

small pit houses + large pit houses

$$42 + 256 = 298$$

I estimate about 298 people lived in the village, using the stated assumptions.

Exemplar #2 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

Assumptions

- Families are the most likely to live in the community so most pit houses will have families.
- Each family requires  $10\text{m}^2$  of living space
- Each pit house has a  $1\text{m}^2$  fire pit

Small pit house

$$A = \pi \left(\frac{3}{2}\right)^2 = 38.48\text{m}^2$$

$$38.48 \times 3 \text{ houses} = 115.45\text{m}^2 - 3(1\text{m}^2) \text{ fire} = 112.45\text{m}^2$$

$$\frac{112.45\text{m}^2}{10\text{m}^2} = \approx 11 \text{ families with } 2.45\text{m}^2 \text{ left over}$$

$$11 \times 4 \text{ people} = 44 \text{ people}$$

Large pit houses

$$A = \pi \left(\frac{11}{2}\right)^2 \quad A = 95.03\text{m}^2$$

$$95.03 \times 8 \text{ houses} = 760.24\text{m}^2$$

$$760\text{m}^2 - 8\text{m}^2 \text{ fire pits} = 752\text{m}^2$$

$$752 \div 10\text{m}^2 = \approx 75 \text{ families}$$

$$75 \times 4 \text{ people} = 300 \text{ people in large houses}$$

$$300 + 44 = 344 \text{ people}$$

I estimate there will be 344 people in the village. This is an estimate because I combined all the living space together, and the actual pit houses might not fit this many, also, there would probably be single adults and couples in the village, and they take more space, so the real numbers probably less than 344.

Exemplar #3 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Incomplete justification (minor error in assuming only adults)
- ✓ Communication is clear, detailed, and organized

Assumptions

- maximum living space for adult =  $4\text{m}^2$
- pit houses have  $1\text{m}^2$  for fire + ladder
- only adults live in pit houses

Small pit house  $A = \pi r^2$

$$A = \pi (3.5)^2$$

$$A = 38.48\text{m}^2$$

$$38.48\text{m}^2 - 1\text{m}^2 \text{ for fire pit} = 37.48\text{m}^2$$

$$\frac{37.48\text{m}^2}{4\text{m}^2} = \sim 9 \text{ people in a small pit house}$$

$$9 \times 3 \text{ houses} = 27 \text{ people}$$

Large pit house:  $A = \pi (5.5)^2$ ,  $A = 95.03\text{m}^2$

$$95.03\text{m}^2 - 1\text{m}^2 \text{ fire} = 94.03\text{m}^2$$

$$\frac{94.03\text{m}^2}{4\text{m}^2} = \sim 23 \text{ people} \quad 23 \times 8 \text{ pit houses} = 184 \text{ people}$$

$184 + 27 = 211$ . There would be about 211 adults in the village, although if there were kids then there would be more because families take less room.

Exemplar #4 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Minor error in mathematical solution (didn't subtract area of fire pit)
- ✓ Incomplete justification (minor error in assuming only adults)
- ✓ Communication is clear

Adults require  $4\text{m}^2$  of living space, 11m diameter houses hold 23 adults, 7m diameter houses hold 9 adults. For a total of 211 Adults that lived in the village.

$\pi r^2 = A$	11m diameter houses
$\pi(\frac{11}{2})^2 = A$	$95\text{m}^2$
$95.0 = A$	$4\text{m}^2 = 23.758 \text{ adults} = 23 \text{ adults}$
$\pi r^2 = A$	7m diameter houses
$\pi(\frac{7}{2})^2 = A$	$38.5\text{m}^2$
$38.5 = A$	$4\text{m}^2 = 9.62 \text{ adults} = 9 \text{ adults}$
8 - 11m	$23 \times 8 = 184 \text{ adults}$
3 - 7m	$9 \times 3 = 27 \text{ adults}$
	<u>211 adults in the village</u>

Exemplar #5 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Incorrect mathematical solution (based on unreasonable assumptions)
- ✓ Incomplete justification (minor error in assuming only adults)
- ✓ Information is not clearly communicated

11 x 8	all adults
7 x 3	no families
	no ladders, fire pits
	or sleeping room
$\pi \left(\frac{11}{2}\right)^2 \times 8 \div 4 = 190$	
$\pi \left(\frac{7}{2}\right)^2 \times 3 \div 4 = 28 = 218 \text{ people}$	

Exemplar #6 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Major calculation error (incorrect use of diameter)
- ✓ Incomplete justification (minor error in assuming only adults)
- ✓ Communication is clear

assumptions

- ① the minimum space for 1 person is  $4m^2$
- ② the space required for ladder and fire pit is neglected (doesn't count).
- ③ the diameter of pit houses are either 11m or 7m exact
- ④  $\pi$  equals to 3.14

total area =  $8 \times \pi \times 11^2 + 3 \times \pi \times 7^2$   
 $= 3501.1m^2$

number of people living in the village =  $\frac{\text{total area}}{\text{minimum spaces}}$   
 $= \frac{3501.1m^2}{4m^2}$   
 $= 875.28$   
 $= 875 \text{ (people)}$

The max number of people is 875.

Exemplar #7 – Score: 1

- ✓ Work shows a limited understanding of the situation
- ✓ Inappropriate strategy selected (area not used)
- ✓ Incorrect mathematical solution
- ✓ No justification
- ✓ Information is not clearly communicated

$11 = \text{houses}$   
 $8 = 11 \text{ m}$   
 $3 = 7 \text{ m}$

$\frac{1}{2}$  of the 11 meter house can fit approx.  
 4 people and the 7m house can fit approx  
 3 people...

$4.8$	$+$	$= 53$	$\text{people (estimate)}$
$3.7$	$\times$	$\text{can fit in this village}$	



## Exemplar #8 – Score: 1

- ✓ Work shows a limited understanding of the situation
- ✓ Inappropriate strategy selected (area not used)
- ✓ Incorrect mathematical solution
- ✓ No justification
- ✓ Communication is clear but incomplete

There are 8 sections that are 11 meters.  
I estimate that 14 people can fit in a  
11 meters of space.

Then there are 3 sections of 7  
meters of space which could probably  
hold 8 people so

$$14 \times 8 = 112 \quad 112 + 24 = 136$$

$$3 \times 8 = 24$$

approximately 136 people live  
in that village.

Five years ago, Jae Eun and Ted, started a company together. They each contributed a different amount of money to start the company.



The Company created several popular video games. Now Jae Eun and Ted receive an offer from an individual who wants to buy their company.

26. Five years after the start of The Company, The Founders decide to sell it for \$750 000. To determine their fair share of the sale price, they agree that any contributions made towards start-up costs will be worth 1.5 times their original value. Contributions made after start-up will not be adjusted.

How much should Jae Eun and Ted each receive from the sale of their company?

Explain and justify your solution.

This question is to be answered on paper.

### **Summary of Requirements for a Level 4:**

- ▶ Determine the current value of the initial investments both partners made (Jae Eun \$87 750 and Ted \$33 000)
- ▶ Calculate the total investment over 5 years (Ted \$108 000 and Jae Eun \$87 750)
- ▶ Select and communicate a strategy for sharing the \$750 000
- ▶ Provide evidence to justify solution
- ▶ Communicate solution in context

There are several possible solutions; a few solutions are given.

### **Solution 1**

This solution is based on each partner receiving a percentage of the total sale price based on what they invested in the company.

The initial investment is worth 1.5 times its original value. Jae Eun and Ted contributed different amounts at the start.

$$\begin{aligned} \text{Jae Eun: } & \$50\,000 + \$8\,500 = \$58\,500 \\ & \$58\,500 \times 1.5 = \$87\,750 \text{ (Jae Eun's initial contribution weighted at } 1.5\times) \end{aligned}$$

$$\text{Ted: } \quad \$22\,000 \times 1.5 = \$33\,000 \text{ (Ted's initial contribution weighted at } 1.5\times)$$

Ted continued to contribute \$1250/month for 5 years.

$$\$1250 \times 5 \text{ years} \times 12 \text{ months/year} = \$75\,000$$

$$\text{In total, Ted invested: } \$33\,000 + \$75\,000 = \$108\,000$$

$$\text{Altogether, Jae Eun and Ted contributed: } \$87\,750 + \$108\,000 = \$195\,750$$

## Video Game Company

The percentage of \$195 750 that each partner contributed:

$$\text{Jae Eun} \quad \frac{\$87\,750}{\$195\,750} \times 100 = 44.8\%$$

$$\text{Ted} \quad \frac{\$108\,000}{\$195\,750} \times 100 = 55.2\%$$

Therefore, Jae Eun should receive 45% (or 44.8%) of the sale price, while Ted should receive 55% (or 55.2%).

Jae Eun:  $\$750\,000 \times 0.45 = \$337\,500$  (or  $\$336\,000$  using decimal %)

Ted:  $\$750\,000 \times 0.55 = \$412\,500$  (or  $\$414\,000$  using decimal %)

## Solution 2

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This solution is based on splitting the profit of the sale after deducting the partners' contributions.

The same initial calculations as in Solution 1:

Jae Eun:  $\$50\,000 + \$8\,500 = \$58\,500$

$\$58\,500 \times 1.5 = \$87\,750$  (Jae Eun's initial contribution weighted at 1.5x)

Ted:  $\$22\,000 \times 1.5 = \$33\,000$  (Ted's initial contribution weighted at 1.5x)

Ted continued to contribute \$1250/month for 5 years.

$\$1250 \times 5 \text{ years} \times 12 \text{ months/year} = \$75\,000$

In total, Ted invested:  $\$33\,000 + \$75\,000 = \$108\,000$

Total Partner contributions:  $\$87\,750 + \$108\,000 = \$195\,750$

Sale price of Company less Total Partner contributions:  $\$750\,000 - \$195\,750 = \$554\,250$

Profit split equally:  $\$554\,250 \div 2 = \$277\,125$  for each partner

Therefore:

Jae Eun:  $\$87\,750 + \$277\,125 = \$364\,875$

Ted:  $\$108\,000 + \$277\,125 = \$385\,125$

### Solution 3

---

This solution is based on splitting the profit of the sale and crediting the difference between investment contributions to the partner that paid more.

The same initial calculations as in Solution 1:

$$\text{Jae Eun: } \$50\,000 + \$8\,500 = \$58\,500$$

$$\$58\,500 \times 1.5 = \$87\,750 \text{ (Jae Eun's initial contribution weighted at } 1.5\times\text{)}$$

$$\text{Ted: } \quad \quad \quad \$22\,000 \times 1.5 = \$33\,000 \text{ (Ted's initial contribution weighted at } 1.5\times\text{)}$$

Ted continued to contribute \$1250/month for 5 years.

$$\$1250 \times 5 \text{ years} \times 12 \text{ months/year} = \$75\,000$$

$$\text{In total, Ted invested: } \$33\,000 + \$75\,000 = \$108\,000$$

$$\text{Ted invested more than Jae Eun: } \$108\,000 - \$87\,750 = \$20\,250.$$

Ted paid \$20 250 more than Jae Eun.

$$\text{Split the sale income: } \$750\,000 \div 2 = \$375\,000$$

As Ted invested more in the company, he will receive \$20 250 more than half the sale price, while Jae Eun will receive \$20 250 less than half the sale price:

$$\text{Jae Eun: } \$375\,000 - \$20\,250 = \$354\,750$$

$$\text{Ted: } \quad \quad \quad \$375\,000 + \$20\,250 = \$395\,250$$

## Video Game Company

### Exemplar #1 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

Joe:  $\$54,500 \times 1.5 = \$81,750$

Ted:  $\$22,000 \times 1.5 = \$33,000 + \$1750 \times 12 \text{ months} \times 5 \text{ years} = 75,000$

Total =  $\$104,000$

plu percentage contributions. total =  $\$195,750$

% Joe =  $\frac{\$81,750}{\$195,750} \times 100 = 44.4\%$

% Ted =  $\frac{\$104,000}{\$195,750} \times 100 = 55.2\%$

---

each compensation:

Joe:  $\$750,000 \times 0.448 = \$336,000$

Ted:  $\$750,000 \times 0.552 = \$414,000$

Joe should receive 44.4% of the money from the sale. Because that is the percent he contributed to the company in funds. This means that Joe should receive  $\$336,000$ .

Ted should receive 55.2% of the money from the sale. The reason for this is because he contributed 55.2% to the company when it comes to money, so in theory he owns 55.2% of the company so he should receive  $\$414,000$  of the  $\$750,000$ .

## Exemplar #2 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

Contributions: J.E.  $\rightarrow (8500 + 50000) \cdot 1.5 = 87750$   
 Ted  $\rightarrow (22000 \cdot 1.5) + (60 \cdot 1250) = 108000$

Contributions combined  $= 87750 + 108000 = 195750$

Percent of total contributions: J.E.  $\rightarrow 87750 \div 195750 = 0.448 \approx 0.45$   
 Ted  $\rightarrow 108000 \div 195750 = 0.551 \approx 0.55$

Jae Eun should receive 0.45 of the sell because J.E. committed 0.45 (45%) of the total amount of money contributed to the company over the course of the five years. Therefore J.E. should receive \$337,500 for selling their company. Ted should receive the remaining 0.55 (55%) of the sell because he committed 55% of the money contributed to the company over the course of the five years. So Ted should receive \$412,500 for selling their company.

Exemplar #3 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear and organized

<p>\$ 750,000</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top;"> <p><u>Ted</u></p> <p>1250 per month</p> <p>22,000 internet x 1.5</p> <hr/> <p>37,000</p> <p>+ 75,000</p> <hr/> <p>108,000 total contributions</p> </td> <td style="width: 50%; vertical-align: top;"> <p><u>Jae</u></p> <p>8500 computer x 1.5</p> <p>50,000 startup x 1.5</p> <hr/> <p>127,500</p> <p>75,000</p> <hr/> <p>87,750 total contributions</p> </td> </tr> </table>	<p><u>Ted</u></p> <p>1250 per month</p> <p>22,000 internet x 1.5</p> <hr/> <p>37,000</p> <p>+ 75,000</p> <hr/> <p>108,000 total contributions</p>	<p><u>Jae</u></p> <p>8500 computer x 1.5</p> <p>50,000 startup x 1.5</p> <hr/> <p>127,500</p> <p>75,000</p> <hr/> <p>87,750 total contributions</p>	<p>Ted should receive \$385,125.</p> <p>Jae should receive \$364,875.</p> <p>I think they should each be paid back what they contributed and then have the rest of the money split in half.</p>	
<p><u>Ted</u></p> <p>1250 per month</p> <p>22,000 internet x 1.5</p> <hr/> <p>37,000</p> <p>+ 75,000</p> <hr/> <p>108,000 total contributions</p>	<p><u>Jae</u></p> <p>8500 computer x 1.5</p> <p>50,000 startup x 1.5</p> <hr/> <p>127,500</p> <p>75,000</p> <hr/> <p>87,750 total contributions</p>			
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## Exemplar #4 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

$$50000 + 8500 = 58500 \times 1.5 = 87750 = \text{Jae}$$

$$22000 + 1250 \times 12 \times 5 = 97000 \times 1.5 = 145500 = \text{Ted}$$

$$750000 - (87750 + 145500) =$$

$$750000 - 233250 = 516750$$

$$516750 \div 2 = 258375$$

Ted and Jae should both get \$258,375 from selling the company as well as 87,750 for Joes contributions and 145,500 for Teds contributions

Exemplar #5 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Correct mathematical solution
- ✓ Incomplete justification
- ✓ Information not clearly communicated

$$50,000 + 8500 = 58,500 \times 1.5 = 87,750$$

$$22,000 \times (1250 \times 12 \times 5) = 97,000 \times 1.5 = 145,000$$

$$\frac{87,750}{145,000} = 60.3\%$$

Ted should gain 60.3% of the company  
 Jae Eun should gain 39.7% of the company

Ted gets \$452,025  
 Jae Eun gets \$297,975

## Exemplar #6 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Minor error in mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear and organized

Sold for 750,000 after 5 years

Jae Eun -  $58,500 \times 1.5 = 87,750$  ( $50,000 + 8,500$ )

Ted -  $97,000$  ( $22,000 + (1250 \times 12 \times 5)$ )

$58,500 + 97,000 = 184,750$  - money put in (total)

Jae Eun =  $\frac{87,750}{184,750} = 0.47$

Ted =  $\frac{97,000}{184,750} = 0.52$  → Their shares of the company

Jae =  $750,000 \times 0.47 = 352,500$

Ted =  $750,000 \times 0.52 = 390,000$

Jae Eun should receive \$352,500 of the profit because he contributed 0.47 of the money to the company. Ted should receive \$390,000 of the profit because he contributed 0.52 of the money to the company.

Exemplar #7 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Incomplete mathematical solution
- ✓ Justification is incomplete
- ✓ Information not clearly communicated

Joe:  $8500 + 50,000 = 58,500 \times 1.05 = \$87,750$   
 Ted:  $22,000 + 1250 / \text{month} = 23,250 \times 1.05 = \$34,875 + 1250 \times 12 \times 5$   
 $34,875 + 75,000 = 109,875$

Joe and Ted should get the same amount which is ~~\$276,875~~ because that is what they contributed through the whole five years they have been running this company.

$$\begin{array}{r}
 750,000 \\
 - 87,750 \\
 - 109,875 \\
 \hline
 552,375 \\
 \hline
 2 \\
 \hline
 \$276,187.5 \text{ each}
 \end{array}$$

Exemplar #8 – Score: 2

- Work shows a basic understanding of the situation
- Inappropriate strategy selected
- Incorrect mathematical solution
- No justification
- Information not clearly communicated

<p>Jae</p> <p>87750</p> <p>108000</p> <p>- 87750</p> <hr style="width: 100%;"/> <p>20250</p>	<p>Ted</p> <p>108000</p> <p>60%</p> <p>450000</p> <p>40%</p> <p>300000</p>	<p>750000</p> <p>2</p> <hr style="width: 100%;"/> <p>375000</p>
<p>Ted gets 450000\$</p> <p>Jae gets 300000\$</p>		

# Video Game Company

## Exemplar #9 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Incomplete mathematical solution and contains errors
- ✓ Incomplete justification
- ✓ Communication is generally clear

<p>Jae Eun</p> <p>\$50,000 cash</p> <p>\$8,500 (computers)</p> <hr/> <p>Total = \$58,500 initial</p> <p style="margin-left: 40px;"><math>\times 1.5</math></p> <hr/> <p>\$87,750</p>	<p>Ted</p> <p>\$22,000 cash</p> <p style="margin-left: 40px;"><math>\times 1.5</math></p> <hr/> <p>\$33,000</p> <p style="margin-left: 40px;">1250 /month <math>\times</math> 5 years</p> <p style="margin-left: 40px;">= 75,000</p>
<p>Total contribution =</p> <p style="margin-left: 40px;">J. E = \$87,750</p> <p style="margin-left: 40px;">Ted = \$108,000</p>	
<p><math>\frac{750,000}{108,000} = \frac{100}{34.4}</math></p>	<p>Jae Eun should receive 40%</p>
<p><math>\frac{750,000}{87,750} = \frac{100}{11.7}</math></p>	<p>Ted 60%</p> <p>at the he contributed more</p>

### Exemplar #10 – Score: 1

- Work shows a limited understanding of the situation
- No strategy implemented
- Incorrect mathematical solution
- Limited justification
- Information is somewhat communicated

I think Jae Eun should make \$87,750 because this is 1.5 times the amount she gave to help the company start (she gave 58500).

Ted should make \$33000 (this is 1.5 times the 22000 ted gave to start the company). Plus, ted should get the \$75000 he has given over the past years. Ted gave 1250 every month for 5 years which adds up to be a total of \$75000. In total ted should get 108000 dollars.

Exemplar #11 – Score: 1

- Work shows a limited understanding of the situation
- No strategy implemented
- Incorrect mathematical solution
- No justification
- Information not clearly communicated

Jae Eun's contribution: 58,500  
 Ted Browns contribution: 22,000

~~$58,500 \times 1.5 = 87,750 + 58,500 = 146,250$~~   
 ~~$22,000 \times 1.5 = 33,000 + 22,000 = 55,000$~~

Jae Eun should recieve 87,750  
 Ted Brown should recieve 33,000



### Exemplar #12 – Score: 1

- Work shows a limited understanding of the situation
- Limited strategy implemented
- Incorrect mathematical solution
- No justification
- Information not clearly communicated

Handwritten work on grid paper:

$$750,000 \div 2 = 375,000$$

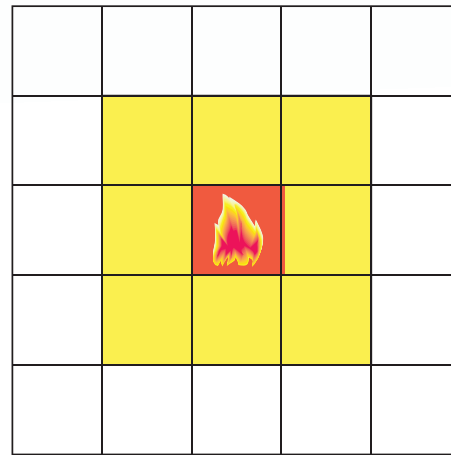
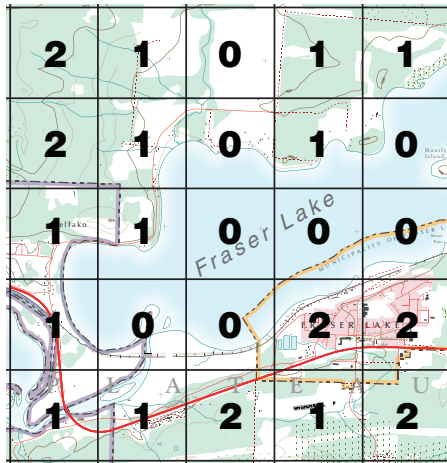
They should get 375,000 dollars each.

You are being trained in managing forest fires. Your Fire Fighting Training Manual describes a forest fire spread simulation.



The map below is a fire grid. It describes the likelihood of a fire spreading to different cells.

Fire may spread from one cell into neighbouring cells as shown:



■ Burning cell    ■ Neighbouring cell

Each cell has the following fire-spread rating reflecting the probability that fire will spread to that cell from a neighbouring cell:

Fire-Spread Rating	Probability of Spreading
<b>0</b>	Fire has a 0% chance of spreading into it from neighbouring cells
<b>1</b>	Fire has a 50% chance, or probability of 0.5, of spreading into it from neighbouring cells
<b>2</b>	Fire has an 80% chance, or probability of 0.8, of spreading into it from neighbouring cells

**Example:** At time zero, the forest in the highlighted cell (🔥) is burning. There are many possible ways that the fire can spread. Two possible scenarios are shown below.



1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1



**Scenario 1**

3 neighbouring cells are burning.



1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1

**Scenario 2**

5 neighbouring cells are burning.

1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1



5 more neighbouring cells are burning.

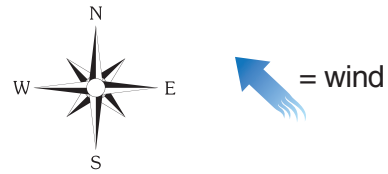


1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1

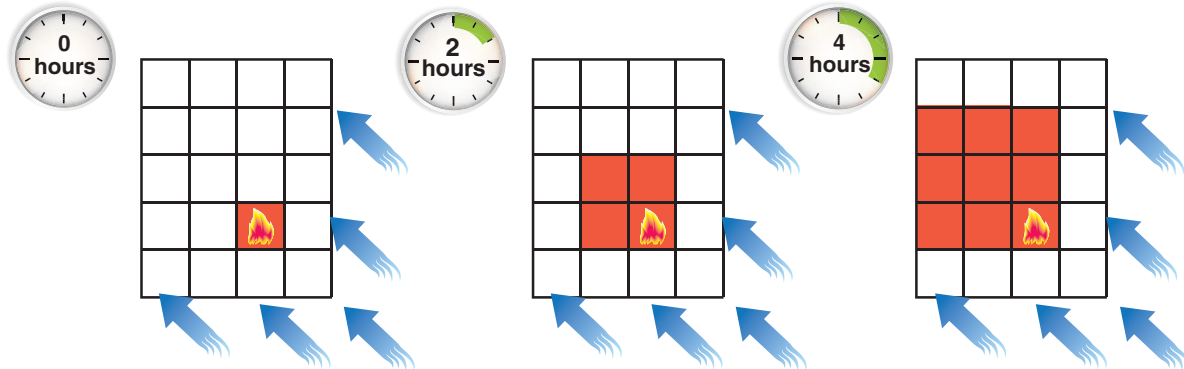
5 more neighbouring cells are burning.

1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1

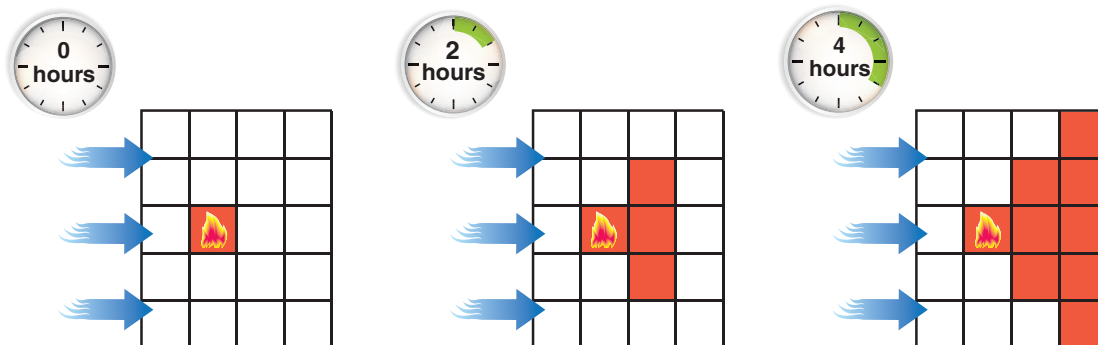
Strong wind conditions can affect the spread of fire into neighbouring cells.



**Wind from the south-east**



**Wind from the west**



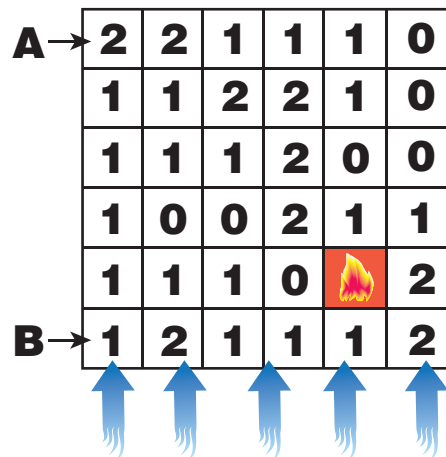
Wind **increases** the probability of fire spreading into neighbouring cells which are **downwind**, and wind **decreases** the probability of fire spreading into neighbouring cells which are **upwind**:

Fire-Spread Rating	Probability of Fire-Spreading		
	Neighbouring (no wind)	Downwind Neighbouring	Upwind Neighbouring
<b>0</b>	0	0	0
<b>1</b>	0.5	0.8	0.2
<b>2</b>	0.8	1.0	0.3

26. At time zero, the highlighted cell is burning and there is a strong, constant wind blowing from the south. There are people living in the areas within cell A and cell B.

What is the minimum time it would take the fire to reach cells A and B?  
 What is the likelihood of the fire spreading to cells A and B within that time?

Explain and justify your solution.



This question is to be answered on paper.

**Summary of Requirements for a Level 4:**

- ▶ Provide time for fire to get to cell A and cell B (minimum 8 hours for both)
- ▶ Likelihood of the fire getting to cell A within that time (within 8 hours, it is high)
- ▶ Likelihood of the fire getting to cell B within that time (within 8 hours, it is very low)
- ▶ Provide evidence to justify solution
- ▶ Communicate solution in context

## Forest Fires

There are several possible solutions; a couple of solutions are given.

### Solution 1

Cell A: The soonest the fire could reach cell A is 8 hours. As cell A is downwind from the highlighted cell, there is a high probability that the fire will spread to that cell in 8 hours (64%).

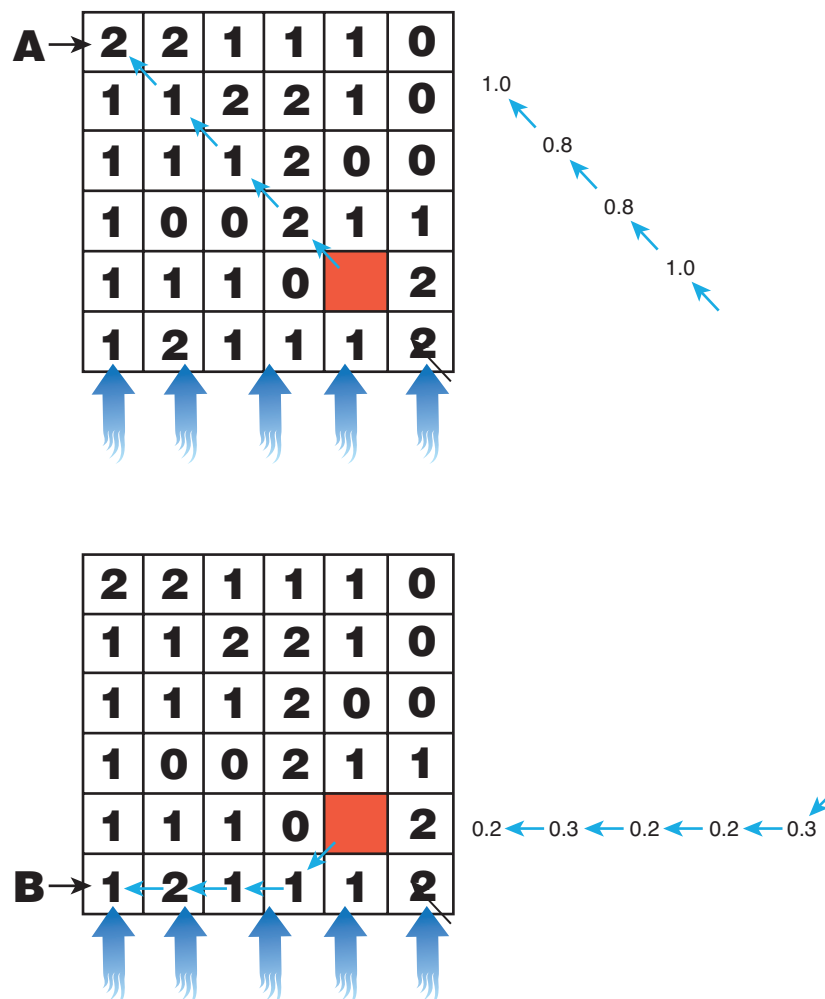
$$\text{Probability} = 1.0 \times 0.8 \times 0.8 \times 1.0 = 0.64$$

Cell B: The soonest the fire could reach cell B is 8 hours, but because it is upwind, there is a low probability (0.24%) that the fire will spread to that cell.

$$\text{Probability} = 0.2 \times 0.2 \times 0.3 \times 0.2 = 0.0024$$

### Solution 2

The fastest the fire could reach both indicated cells is 8 hours. However, the fire is more likely to spread to cell A as it is downwind, and the most probable path is indicated by the grid below. The fire is not likely to spread to cell B because it is upwind. The most probable path is indicated by the grid below.



## Forest Fires

### Note:

There are 6 ways the fire could reach cell B in 8 hours, and the actual probability of spreading to cell B in that time is the sum of all of those cases ( $P = 0.028$ ). It is sufficient for students to provide one case and give that probability.

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0		2
B1	2	1	1	1	2

$$(0.2)^3(0.3) = 0.0024$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0		2
B1	2	1	1	1	2

$$(0.2)^4 = 0.0016$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0		2
B1	2	1	1	1	2

$$(0.2)^3(0.3) = 0.0024$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0		2
B1	2	1	1	1	2

$$(0.2)^4 = 0.0016$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0		2
B1	2	1	1	1	2

$$(1.00)(0.2)^3 = 0.008$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0		2
B1	2	1	1	1	2

$$(1.00)(0.2)^2(0.3) = 0.012$$

Exemplar #1 – Score: 4

- ✓ Work shows an advanced understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed, and organized

for the people living in cell A, they have a minimum of 8 hours before the fire arrives. Based on this path:

The odds of it reaching A in 8 hours is 64% based on this calculation:  $(1.0 \cdot 0.8 \cdot 0.8 \cdot 1.0) = 64\%$

Should the fire not arrive in 8 hours it will have an 80% chance every 2 hours to arrive based on this path and this calculation:  $(1.0 \cdot 1.0 \cdot 1.0 \cdot 0.8)$

As for B, the most direct path will arrive in a minimum of 8 hours as well, however the chance is only 0.24% based on the calculation:  $(0.2 \cdot 0.2 \cdot 0.3 \cdot 0.2)$



Exemplar #2 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy implemented
- ✓ Correct mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear, detailed and organized

A is ~~upwind~~ downwind  
 Fire could reach A in 8 hours  
 The probability is  
 $(1.0)(0.8)(0.8)(1.0) = 0.64$

A				
1.0				
	0.8			
		0.8		
			1.0	
				.
B	0.2	0.3	0.2	0.2

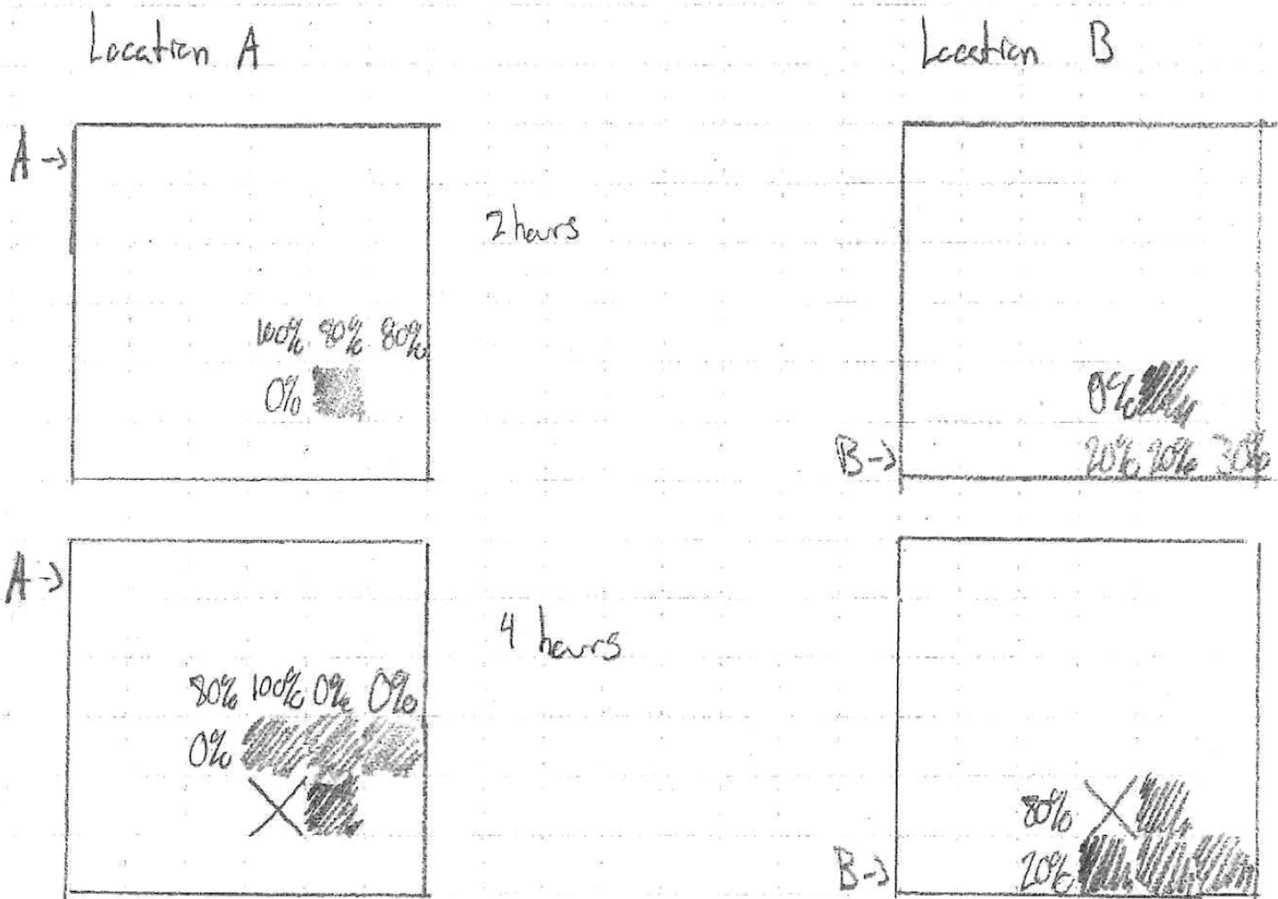
B is ~~downwind~~ upwind  
 Fire could reach B in  
 8 hours.  
 The probability is  
 $(0.2)(0.2)(0.3)(0.2) = 0.0024$

↑ ↑ ↑ ↑ WIND

The probability that the fire reaches A in 8 hours is 64%, which is fairly high.  
 The fire can reach B in 8 hours, but the probability is very low, 0.0024 which is not very likely.

Exemplar #3 – Score: 3

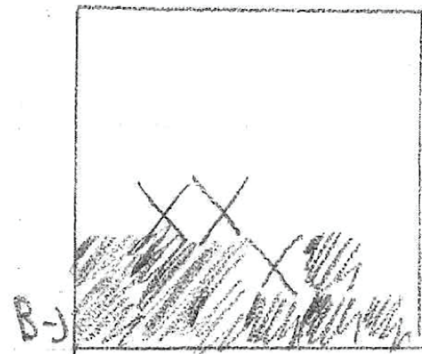
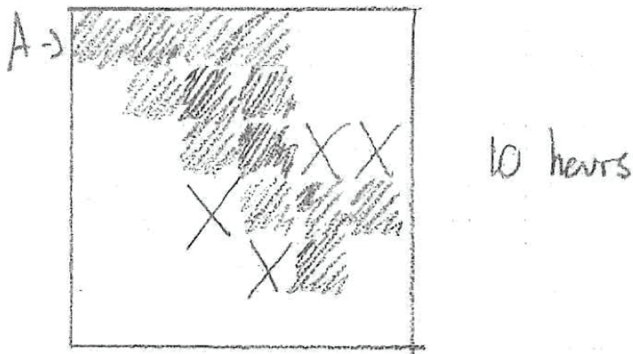
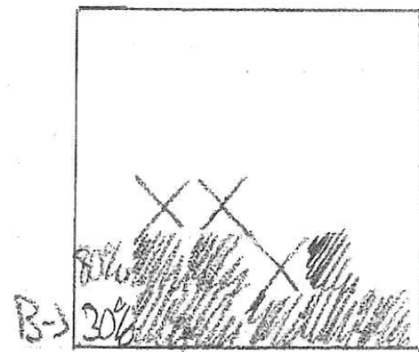
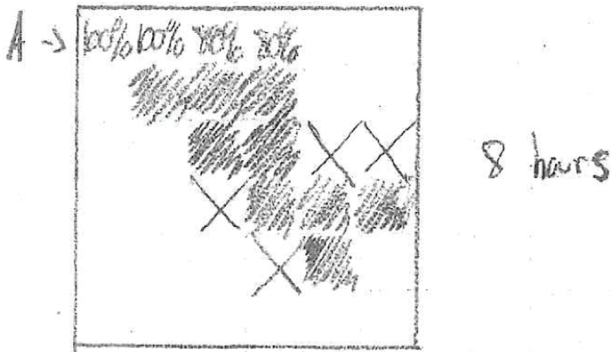
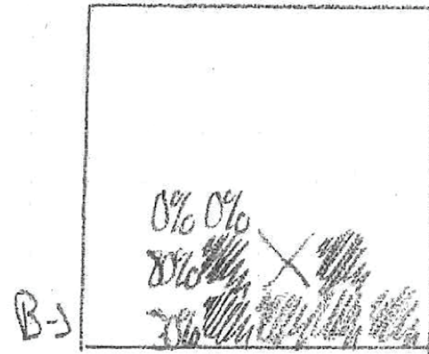
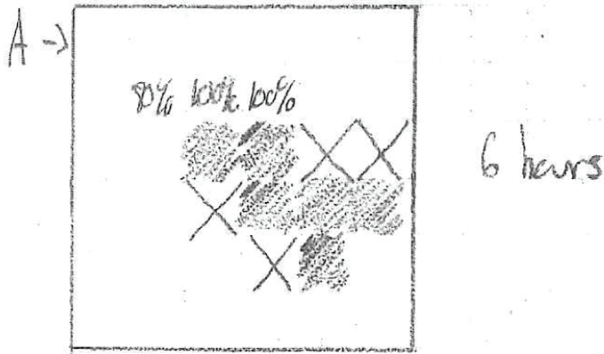
- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Minor error in mathematical solution
- ✓ Solution is reasonable within the assumptions made
- ✓ Communication is clear



continued on next page

Location A

Location B



The fire will reach location A in 10 hours. The people should evacuate as the fire will arrive soon. The probability is 100%.

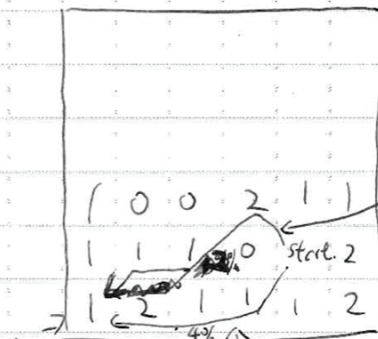
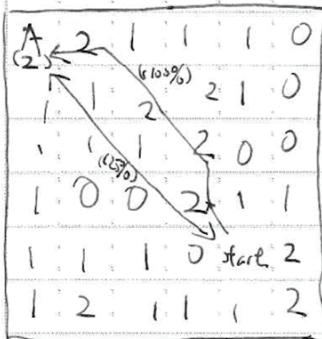
Worst case the fire the fire will reach location B in 10 hours, however the probability is 0.24%. So the people are probably safe.

Exemplar #4 – Score: 3

- ✓ Work shows an effective understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Minor error in mathematical solution
- ✓ Solution is reasonable within the assumptions made
- ✓ Communication is clear

cell A Assuming worst case scenario.  
 A → If it spreads straight diagonal, they have 8 hours to leave.  
 But there will be a 25% chance of it spreading to them in 8 hours.  
 $1 \times 0.5 \times 0.5 \times 1 = 0.25 = 25\%$   
 however, if we follow the path of cells marked with 2. (100% chance due to the wind) then it would take ~~at~~ 10 hours to spread to the cell A indefinitely.

Cell B. at least 8 hours, and because they are up wind, the highest possible chance is either.  $0.2 \times 0.5 \times 0.5 \times 0.5 = 0.04 = 4\%$   
 $1 \times 0.2 \times 0.5 \times 0.2 = 0.02$  or  $0.2\%$  (possibility).  
 So there is a very minimal chance that cell B would catch on fire.



I think this route has the highest probability due to taking cells marked with 2 twice, and one on which is downwind.

But this takes one less upwind cells.

## Forest Fires

### Exemplar #5 – Score: 2

- ✓ Works shows a basic understanding of the situation
- ✓ Inappropriate strategy selected
- ✓ Incorrect mathematical solution
- ✓ Mathematical solution is not explained in context
- ✓ Communication is generally clear

Upwind - blank neighbourhoods // boxes  $\Rightarrow$  decreasing probability of fire due to wind

Downwind - shaded neighbourhoods / boxes / cells  $\Rightarrow$  increase probability of fire due to wind

2 hours  $\rightarrow$  8 cells shaded

4 hours  $\rightarrow$  16 cells shaded

5 hours  $\rightarrow$  20 cells shaded

In cell B - they will have a long time as the wind is blowing and it's an upwind which means it's a decreasing probability of fire due to wind

4 hours  $\rightarrow$  16 cells shaded

x hours  $\rightarrow$  20 cells shaded

80 cells =  $16x \div \frac{1}{16}$   
 $x = 5$  hours

Possibly, as mentioned, it's a strong, constant wind blowing.

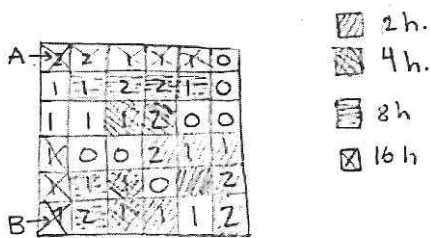
It will take about 5 hours in order for Cell A to get affected due to fire.

Moreover cell A has a fire-speed rating of 2, meaning that it has an 80% / 0.8 probability of neighbouring fire spreading or downwind 1.0.

On the other hand cell B, has a 0.5 / 50% probability of neighbouring fire spreading and a 0.8 downwind.

Exemplar #6 – Score: 2

- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected but implemented incorrectly
- ✓ Incorrect mathematical solution
- ✓ Evaluates mathematical solution in context
- ✓ Communication is clear



In the worst case scenario,  
 \* A cell people have 16 hours before they are in danger  
 \* B cell people also have 16 hours before they are in danger.

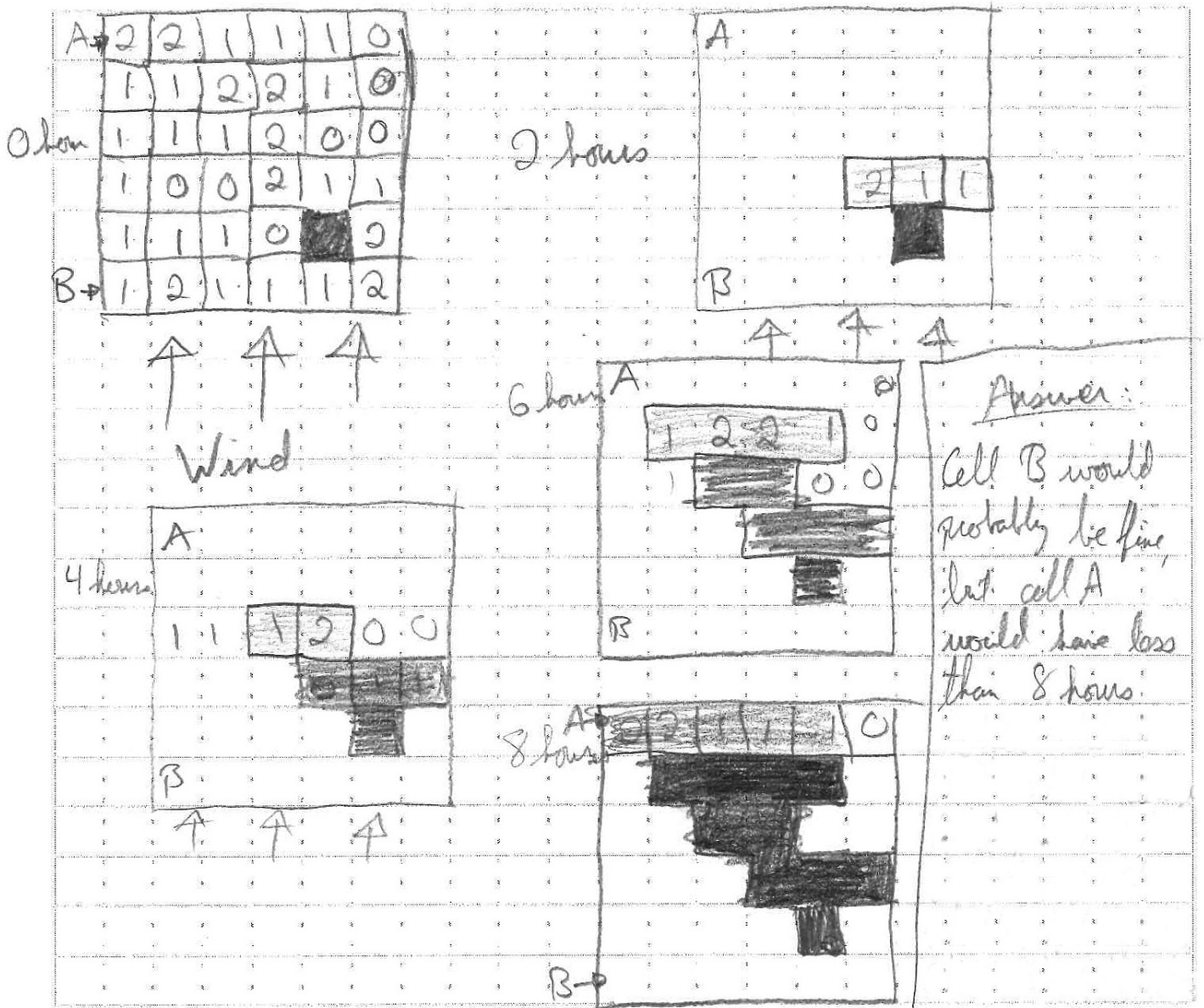
Since the wind is pointing north towards A cell, it is downwind, so A cell has a higher probability of burning within the 16 hours the fire would take to spread in the worst case scenario.

People in cell B could be more relaxed because its not very likely that the fire reaches cell B in the 16 hours, for B cell is upwind.

# Forest Fires

## Exemplar #7 – Score: 2

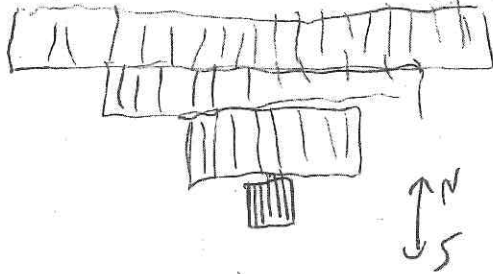
- ✓ Work shows a basic understanding of the situation
- ✓ Appropriate strategy selected
- ✓ Incomplete mathematical solution
- ✓ No justification
- ✓ Communication is generally clear



Exemplar #8 – Score: 1

- Work shows a limited understanding of the situation
- No strategy used
- Incomplete mathematical solution
- Incomplete justification
- Information is somewhat communicated

The people in cell A ought to worry about the fire in 8 hours. This is because of the following pattern:



Luckily, the residents of town B should be spared any damage because they are upwind and it's very difficult for fire to spread upwind.



### Exemplar #9 – Score: 1

- Work shows a limited understanding of the situation
- No strategy used
- Incomplete mathematical solution
- Incomplete justification
- Information is somewhat communicated

Person A has at least 8 hours before the fire will spread to them in the worst case scenario. The wind is blowing towards him, so the fire is extremely likely to spread to him.

Person B probably has upwards of 16 hours to act. He is behind the fire always, so he has a very small chance of the fire spreading to him.

Exemplar #10 – Score: 1

- ✓ Work shows a limited understanding of the situation
- ✓ No strategy used
- ✓ Incorrect mathematical solution
- ✓ Incomplete justification
- ✓ Information not clearly communicated

The people who live in Area A don't have much time because the wind is going south. The people in Area B have more time because the wind is going the opposite way.

